10/3

UNIT-2

Non-Uniform Flow in open channels

Intro :- As Explained in previous chapter the non-uniform flow or varied flow in a channel is the one in which depth of flow changes from section to section along the length of the channel.

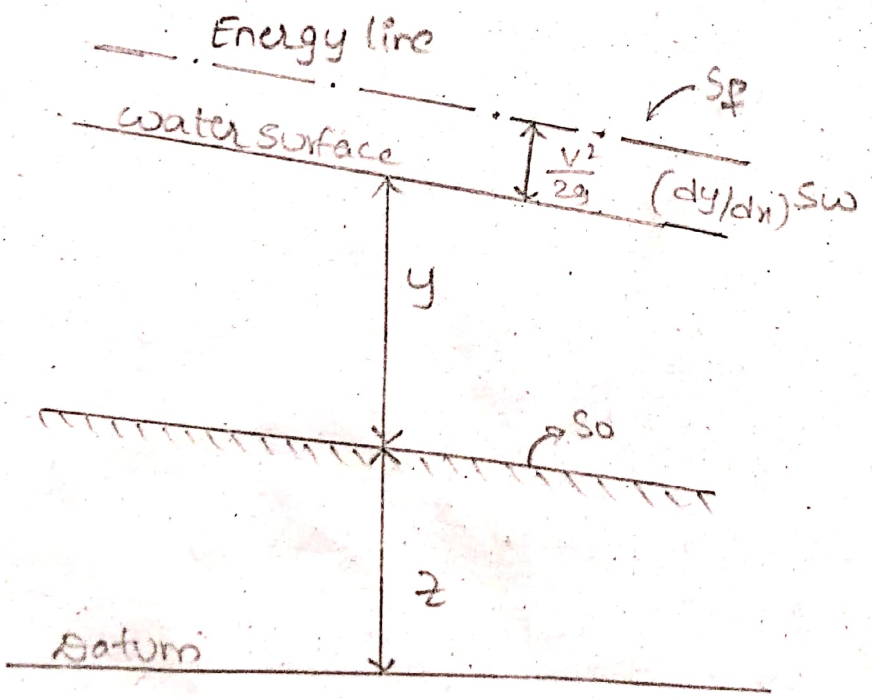
It is furtherly classified as

1. Gradually varied flow (GVF)
2. Rapidly varied flow (RVF)

* The problem of Gradually varied flow is that of predicting overall flow pattern, in other words prediction of the water surface profile to be expected in a given chann-el with human ~~studying~~ steady discharge.

*

Dynamic equation of Gradually Varied flow :-



$$V = \frac{1}{n}(R)^{2/3}(S)^{1/2}_{(GVf)} \Rightarrow (S_f)_{GVf} = \frac{Vn}{R^{2/3}}$$

$$V = C\sqrt{RS_{(GVf)}} \Rightarrow (S_f)_{GVf} = \left(\frac{V}{C\sqrt{R}}\right)^2$$

$$H = \frac{V^2}{2g} + y + z$$

$$H = \frac{Q^2}{2gA^2} + y + z \qquad \left[\because \bar{A}^{2} = -2A^{-3}\right]$$

$$\frac{dH}{dx} = \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right) + \frac{dy}{dx} + \frac{dz}{dx} \qquad \left[\frac{1}{A^2}\right.$$

$$\frac{dH}{dx} = -\frac{Q^2}{gA^3}\left(\frac{dA}{dx}\right) + \frac{dy}{dx} + \frac{dz}{dx}$$

$$-S_f = -\frac{Q^2}{gA^3}\left(\frac{dA}{dx}\right) + \frac{dy}{dx} - S_0$$

we know, $\frac{dA}{dx} = \frac{dA}{dy} * \frac{dy}{dx} = T \cdot \frac{dy}{dx}$

$$\therefore -S_f = -\frac{Q^2 T}{gA^3}\frac{dy}{dx} + \frac{dy}{dx} - S_0$$

$$* \boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}}}$$

$$\frac{Q^2 T}{gA^3} = F_r^2$$

The above eqn is the basic differential eqn for the gradually varied flow.

It is observed that

* $S_0 = S_f \Rightarrow \frac{dy}{dx} = 0$ – uniform flow.

$S_0 \neq S_f \Rightarrow \frac{dy}{dx} \neq 0$ – Non uniform flow.

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→ when $\frac{dy}{dx}$ is positive the water surface is raising. and when $\frac{dy}{dx}$ is (negative) the water surface is fallen.

→ Dynamic eqn for GVF in wide rectangular channel :-

*

$$R = \frac{By}{B+2y} = \frac{By}{B} = y$$

further acceleration according to mannings formula.

$$Q = \frac{1}{n}(By)(y)^{2/3}(S_f)^{1/2}$$

$$= \frac{1}{n}(By_n)(y_n)^{2/3}(S_0)^{1/2}$$

Hydraulic depth is replaced with depth of flow. & n is same for uniform & non-uniform

depth of flow is known as y_n = normal depth.

$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^{10/3}$$

If chezy's formula is used

$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^3$$

In the same manner for channel of rectangular sec.

$$\frac{Q^2T}{gA^3} = \frac{Q^2B}{g(B^3y^3)} = \frac{q^2}{gy^3} = \left(\frac{y_c}{y}\right)^3$$

$$\boxed{\frac{dy}{dx} = S_0 \frac{\left(1-\frac{y_n}{y}\right)^{10/3}}{1-\left(\frac{y_c}{y}\right)^3}}$$

&

$$\boxed{\frac{dy}{dx} = S_0 \frac{1-\left(\frac{y_n}{y}\right)^3}{1-\left(\frac{y_c}{y}\right)^3}}$$

13/3

→ Relation b/w water surface profile and slopes channel bottom slopes :-

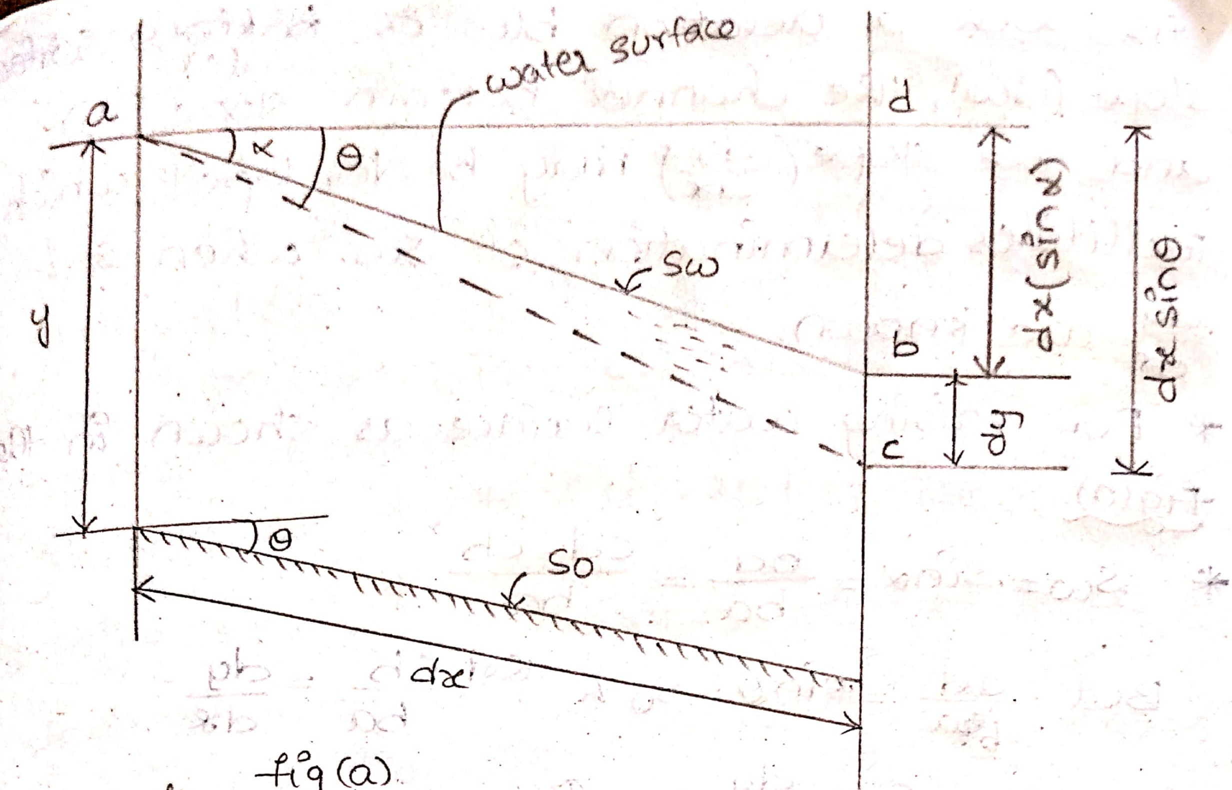


fig (a).

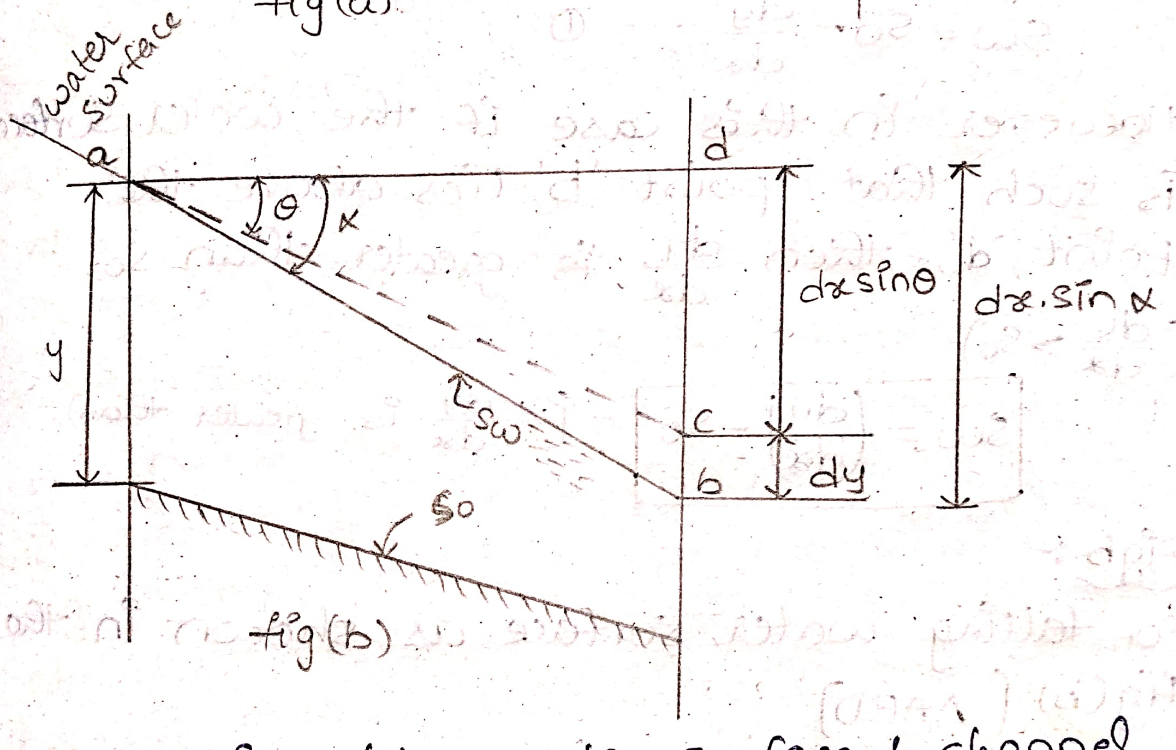


fig (b)

Relation b/w water surface & channel bottom.

* the term "$\frac{dy}{dx}$" represents the slope of the bottom surface with respected to the channel bottom. But often the water surface profile (or) slope (Sw) with respect to horizontal may be required to be determined.

As such a relation b/w a bottom surface water slope (S_w), the channel bottom slope (S_0) and the slope ($\frac{dy}{dx}$) may be developed which facilitates determination of S_w. when S_0 & $\frac{dy}{dx}$ are known.

* For raising water surface as shown in the fig (a).

$$* \quad S_w = \sin\alpha = \frac{bd}{ba} = \frac{cd - cb}{ba}$$

$$\text{But } \frac{cd}{ba} = \sin\theta = S_0 \ \& \ \frac{cb}{ba} = \frac{dy}{dx}$$

$$S_w = S_0 - \frac{dy}{dx} \quad \text{—} \ ①.$$

However in this case if the water surface is such that point 'b' lies above the point 'd', then $\frac{dy}{dx}$ is greater than S_0 ($\frac{dy}{dx} > S_0$)

$$\boxed{S_w = \left(\frac{dy}{dx}\right) - S_0} \quad [\because \tfrac{dy}{dx} \text{ is greater than}]$$

Fig b :-

For falling water surface as shown in the fig (b) [ΔABD]

$$S_w = \sin\alpha = \frac{bd}{ba} = \frac{cd + cb}{ba}.$$

$$\text{Again } \frac{cd}{ba} = \sin\theta = S_0, \ \frac{cb}{ba} = \frac{dy}{dx}$$

$$S_w = S_0 + \frac{dy}{dx}.$$

→ classification of channel bottom slopes :-

1. critical slope
2. Mild slope
3. Steep slope

4. Horizontal slope.
5. Adverse slope.

* **Critical slope** :- The channel bottom slope is designated as critical when the bottom slope (S_0) is equal to the critical slope (S_c). [$S_0 = S_c$], thus in the case of normal depth of flow will be equal to the critical depth. [$y_n = y_c$]

* **Mild slope** (M) :- The channel bottom slope is designated as mild when the bottom slope (S_0) is less than the critical slope (S_c) [$S_0 < S_c$] the application of manning's or chezy's formulae will indicate that when the bottom slope is ~~ω~~ mild, the normal depth of flow is greater than critical depth. [$y_n > y_c$]

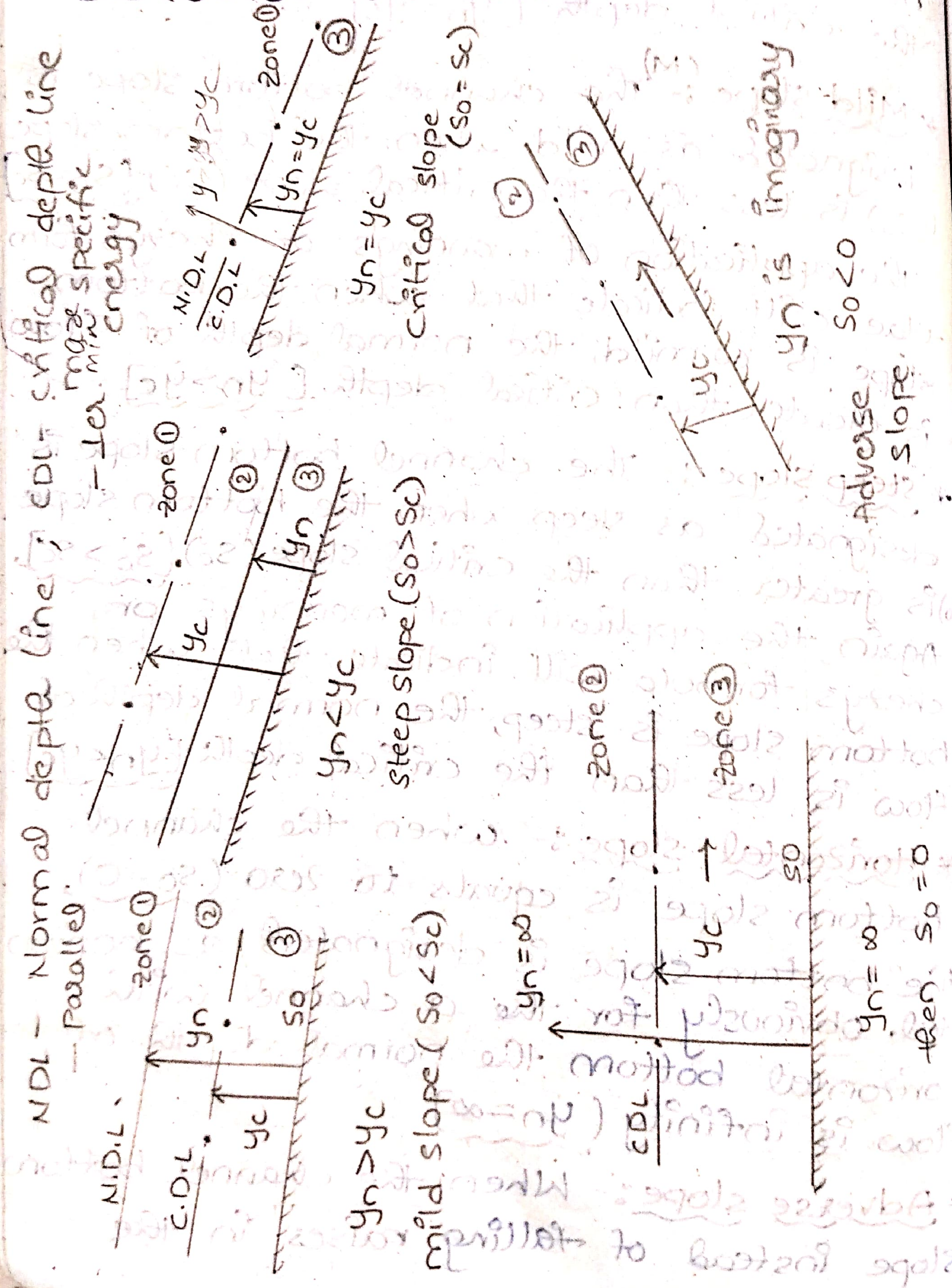
* **Steep slope** :- The channel bottom slope is designated as steep when the bottom slope (S_0) is greater than the critical slope (S_c) [$S_0 > S_c$]. Again the application of manning's or chezy's formula will indicate that when the bottom slope is steep, the normal depth of flow is less than the critical depth [$y_n < y_c$]

* **Horizontal slope** :- when the channel bottom slope is equals to zero ($S_0 = 0$), the bottom slope is designated as horizontal. obviously for the a channel with horizontal bottom the normal depth of flow is infinity ($y_n = \infty$)

* **Adverse slope** :- When the channel bottom slope instead of falling raises in the

direction of flow. it is designated as adverse slope. thus in a channel with adverse ~~water~~ bottom slope [$S_0 < 0$] or it is negative -ve. obviously for an adverse sloped channel the normal depth of flow 'y_n' is imaginary or it is not exist.

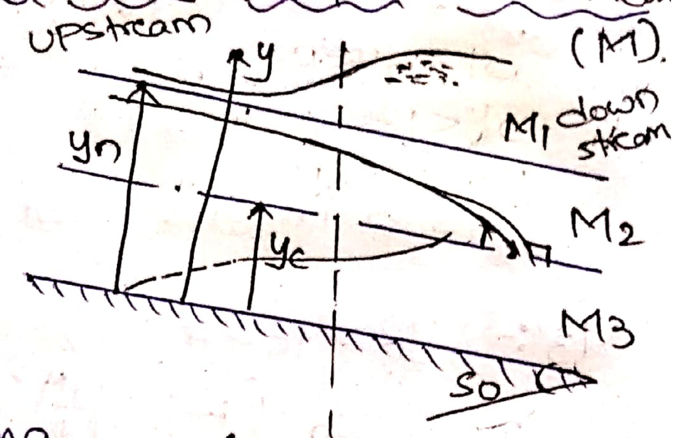
→ classification of surface profiles :-



16/3

characteristics of surface profiles :-

1. surface profile in mild sloped channels: (M)



$(y_n > y_c)$
$(S_o < S_c)$

M_1 :- $y > y_n > y_c$.

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y\uparrow}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^3}$$

$$= \frac{+ve}{+ve} = + \text{ value}$$

- water profile is rising

upstream :- $y = y_n$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y_n}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^{\downarrow}_{\uparrow}} = 0$$

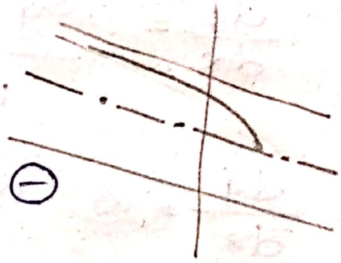
$\frac{dy}{dx} = 0$, means water profile is llel to NDL

downstream :- $y = \infty$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{\infty}\right)^3\right]}{1-\frac{y_c}{\infty}} = S_o$$

$\frac{dy}{dx} = S_o$ means water profile is llel to So.

M_2 :- $y_n > y > y_c$



$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n\uparrow}{y\downarrow}\right)^3\right]}{1-\left(\frac{y_c\downarrow}{y\uparrow}\right)} = \frac{-}{+} = -$$

- water profile is falling in zone 2

upstream :- $y_n = y$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\frac{y_c\downarrow}{y\uparrow}} = \frac{0}{+} = 0$$

$\frac{dy}{dx} = 0$ means water profile is llel to NDL.

downstream :- $y = y_c$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n\uparrow}{y\downarrow}\right)^3\right]}{1-\left(\frac{\cancel{y_c}}{\cancel{y_c}}\right)} = \frac{\ominus S_o}{0} = \infty$$

$\frac{dy}{dx} = \infty$ means $\perp$er to zone-② (CDL)

M3 :- $y_n > y_c > y$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n\uparrow}{y\downarrow}\right)^3\right]}{1-\left(\frac{y_c\uparrow}{y\downarrow}\right)} = \frac{\ominus}{\ominus} = \oplus$$

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water profile is rising.

upstream :- $y = 0$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right) \frac{y_n}{0}}{1-\left(\frac{y_c}{y}\right)^3 \frac{y_c}{0}\ \infty} = \frac{\infty}{\infty} = \infty$$

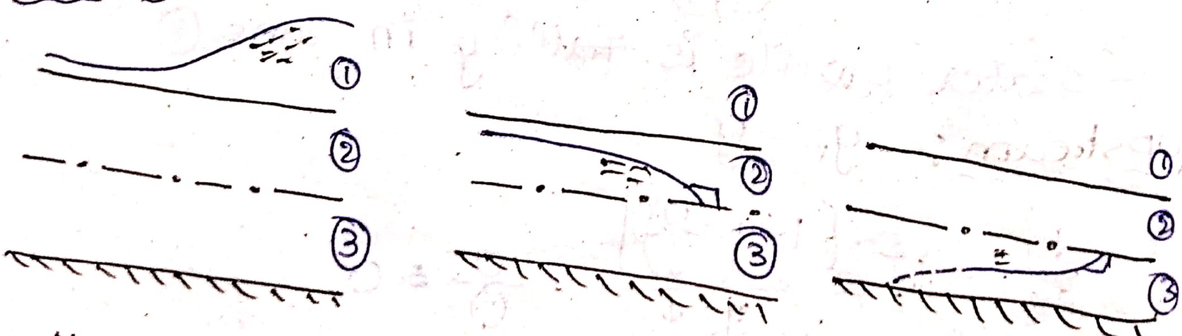
$\frac{dy}{dx} = \infty$ (or) undefined means not $\parallel$el nor $\perp$er.

downstream :- $y = y_c$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\frac{\cancel{y_c}}{\cancel{y_c}}} = \frac{S_o}{0} = \infty$$

$\frac{dy}{dx} = \infty$ means $\perp$er to CDL

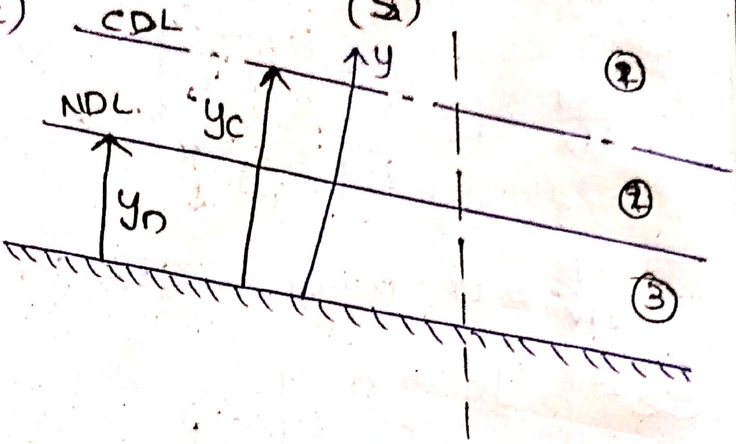
Mild slope :-



$y > y_n > y_c$ $\qquad$ $y_n > y > y_c$ $\qquad$ $y_n > y_c > y$

2. Surface profile in steep sloped channel :- (S)

$(y_n < y_c ;\ S_o > S_c)$



S_1 :- $y > y_c > y_n$.

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^3}$$

$$= \frac{\oplus}{\oplus} = \oplus$$

$\frac{dy}{dx}$ water profile is rising.

upstream :- $y = y_c$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = \frac{\oplus}{0} = \infty$$

$\frac{dy}{dx} = \infty$ means ⊥er to CDL.

downstream :- $y = \infty$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^3} = \frac{S_o \oplus}{\oplus} = S_o$$

$\frac{dy}{dx} = S_o$ means ∥el to S_o.

S_2 :- $y_c > y_i > y_n$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = \frac{S_o \oplus}{\ominus} = \ominus$$

$\frac{dy}{dx} = \ominus$ water profile is falling

upstream :- $y = y_c$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = \frac{\oplus}{0} = \infty$$

$\frac{dy}{dx} = \infty$ means ⊥er to CDL downstream :- $y = y_n$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^{3\uparrow}_{\downarrow}} = \frac{0}{\ominus} = 0$$

$\frac{dy}{dx} = 0$ means ||el to NDL

S_3 :- $y_c > y_n > y$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^{3\uparrow}_{\downarrow}\right]}{1-\left(\frac{y_c}{y}\right)^{3\uparrow}_{\downarrow}} = \frac{\ominus}{\ominus} = +ve$$

$\frac{dy}{dx}$ water profile is rising

upstream :- $y = 0$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right] \frac{y_n}{0}}{1-\left(\frac{y_c}{y}\right)^3 \frac{y_c}{0}} = \frac{\infty}{\infty} = \infty$$

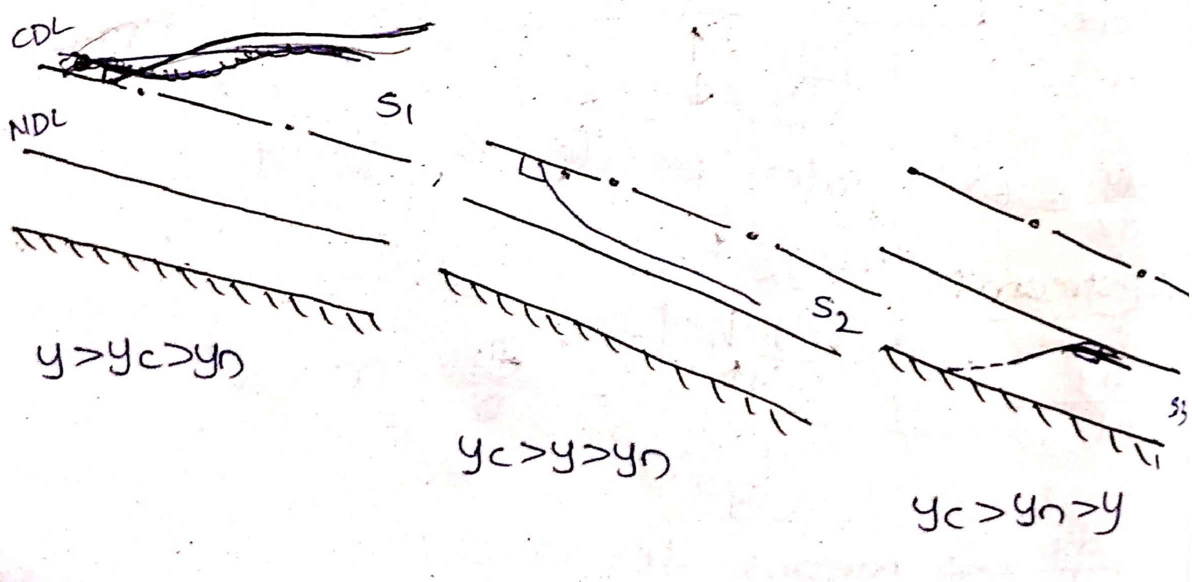
$\frac{dy}{dx} = \infty$ neither perpendicular nor parallel.

downstream :- $y = y_n$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = \frac{S_o}{a} \; 0$$

$\frac{dy}{dx} = 0$ ||el to NDL.

steep slope :-



13

+ Critical ~~Flow~~ slope :- ($y_n = y_c$)($S_o = S_c$) :-

C_1 :- $[y > y_c = y_n]$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^3} = \frac{\oplus}{\oplus} = \oplus$$

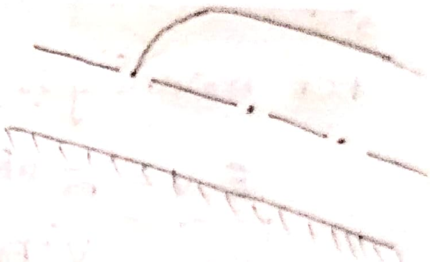
water profile is rising.

upstream :- $y = y_n$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = 0$$

$\frac{dy}{dx} = 0$ neither llel nor ler.

downstream :- $y = \infty$



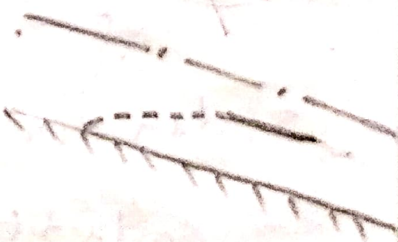
$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n}{y}\right)^3\right]}{1-\left(\frac{y_c}{y}\right)^3} = S_o = S_c$$

$\frac{dy}{dx} = \infty$ llel to NDL

C_3 :- $y_n = y_c > y$

$$\frac{dy}{dx} = \frac{S_o\left[1-\left(\frac{y_n\uparrow}{y\downarrow}\right)^3\right]}{1-\left(\frac{y_c\uparrow}{y\downarrow}\right)^3} = \frac{\ominus}{\ominus} = \oplus$$

water profile is rising



upstream :- $y = 0$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right)}{1-\left(\frac{y_c}{y}\right)^3} = \infty$$

$\frac{dy}{dx} = \infty$ neither llel nor ler

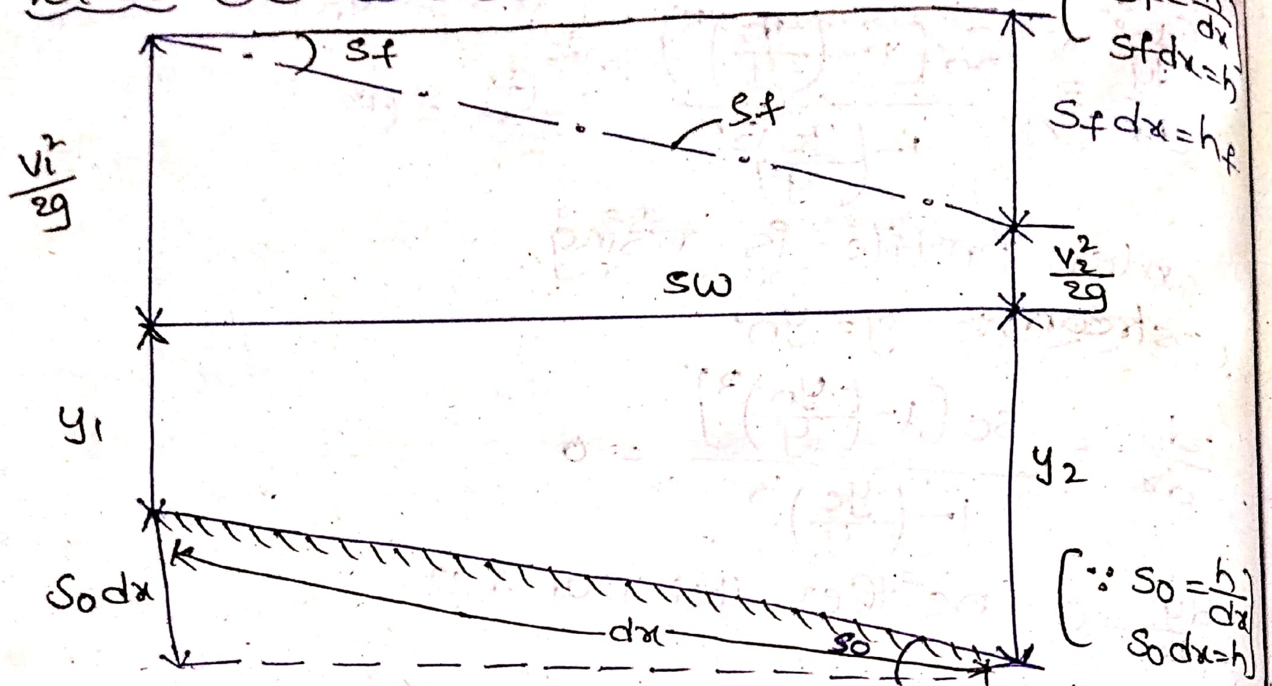
downstream :- $y = y_n = y_c$

$$\frac{dy}{dx} = \frac{S_o\left(1-\left(\frac{y_n}{y}\right)^3\right]\frac{y_n}{y_n}}{1-\left(\frac{y_c}{y}\right)^3\frac{y_c}{y_c}} = 0$$

$\frac{dy}{dx} = 0$ means llel to ND.

→ Integration of the varied flow equation:

Direct step method :-



For manning's,

$y_n = 2.97 \times 1.0 = 2.97$

29.7

$$V = \frac{1}{n} (R)^{2/3} (S_f)^{1/2}$$

$$\boxed{S_f = \frac{V^2 n^2}{(R)^{4/3}}}$$

For chezy's,

$$V = C\sqrt{R S_f}$$

$$S_f = \frac{V^2}{C^2 R}$$

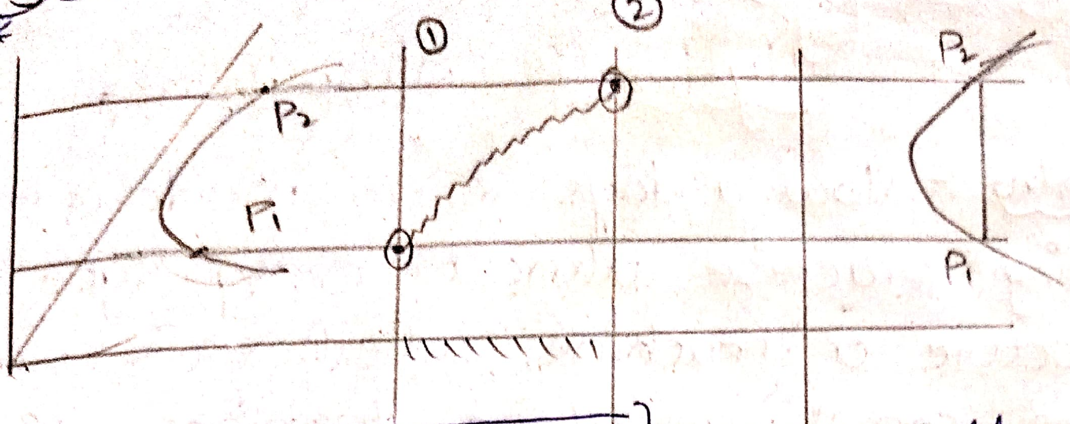
$$S_o\,dx + y_1 + \frac{V_1^2}{2g} = S_f\,dx + \frac{V_2^2}{2g} + y_2$$

$$S_o\,dx + E_1 = S_f\,dx + E_2$$

$$\boxed{dx = \frac{\Delta E}{S_o - S_f}} = \frac{E_1 - E_2}{S_o - S_f}$$

$$\bar{S_f} = \frac{S_{f_1} + S_{f_2}}{2}$$

Hydraulic Jump :-



$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1+\frac{8q^2}{gy_1^3}}\right]$$

$\left[q = \frac{\text{Discharge}}{\text{unit width}} = \frac{Q}{B} \right]$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1+8Fr_1^2}\right]$$

$$Fr_1 = \frac{q}{\sqrt{gy_1^3}} \quad ; \quad Fr_2 = \frac{q}{\sqrt{gy_2^3}}$$

$$\boxed{\Delta E = \frac{(V_1-V_2)^3}{2g[V_1+V_2]}}$$

[Difference specific energy]

$$\Delta E = \frac{(y_2-y_1)^3}{4y_1y_2}$$